**Lecture Sheet**

**On**

**Series**

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**Sequence and Series:** A sequence is defined as an arrangement of any objects or a set of numbers in a particular order followed by some rule. On the other hand, a series is defined as the sum of the elements of a sequence.

If  is a sequence, then the corresponding series is given by



**Note:**  The series is finite or infinite depending if the sequence is finite or infinite.

**Types of Sequence and Series:** Some of the most common examples of sequences and series are:

**Arithmetic Sequences and Series:** A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence. The series consists of the terms of arithmetic sequence is called a series in arithmetic progression. If is an arithmetic sequence, then the corresponding series is given by



**Geometric Sequences and Series:** A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence. The series consists of the terms of geometric sequence is called a series in geometric progression. If is a geometric sequence, then the corresponding series is given by



**Harmonic Sequences and Series:** A sequence in which terms are reciprocal of the terms of an arithmetic sequence is called a harmonic sequence. The series consists of the terms of harmonic sequence is called a series in harmonic progression. If is a harmonic sequence, then the corresponding series is given by



**Fibonacci sequence and Series:** Fibonacci numbers form an interesting sequence of numbers in which each element is obtained by adding two preceding elements and the sequence starts with 0 and 1. Sequence is defined as, F0 = 0 and F1 = 1 and Fn = Fn-1 + Fn-2.

If  is a Fibonacci sequence, then the corresponding series is given by



**Note:**

1. **.**
2. **.**
3. **.**
4. **.**
5. ****
6. **.**

**Problem-01:** Sum the seriesto *n* terms.

**Solution:** Let 











.

**Problem-02:** Sum the series ****.

**Solution:** Let .

Now consider the following identity to find the value of :



Substituting  in (1), we get









.

Adding these we get













.

**Problem-03:** Sum the series ****.

**Solution:** Let .

Now consider the following identity to find the value of :



Substituting  in (1), we get









.

Adding these we get

  












.

**Summation by Method of Difference:** If we are able to express  in the form , where  is some function of , then we can sum the series to  terms.

For, by hypothesis,













whence by addition

.

**Note:** To determine, Firstly, multiply , by the subtraction of the previous term of first term of  from the next term of last term of and then divide the subtraction by an appropriate constant for making equal to .

**Problem-04:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,









where  and .

For , we get .

Therefore, the sum of the given series is



.

**Problem-05:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,









where 

and .

For , we get .

Therefore, the sum of the given series is



. **ANS.**

If in an arithmetic series, every term contains *r* factors then the sum of this series will be determined as follows:

Let the nth term of an arithmetic series is,

 where are constants.





where  and .

Now putting  , we get













whence by addition

 where .

.

**Problem-06:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,



The sum of the given series is





For , we get

.



Putting the value of *c* in (1), we get

 **ANS.**

**Problem-07:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,













The sum of the given series is





For , we get

.



Putting the value of *c* in (1), we get

 **ANS.**

**Problem-08:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,









The sum of the given series is





For , we get

.



Putting the value of *c* in (1), we get





 **ANS.**

**Problem-09:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,







The sum of the given series is





For , we get

.



Putting the value of *c* in (1), we get





 **ANS.**

Again, if in an arithmetic series, every term contains *r* reciprocal factors then the sum of this series will be determined as follows:

Let the nth term of an arithmetic series is,

 where are constants.





where  and 

Now putting  , we get













whence by addition

 where .

.

**Problem-10:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,







where 

and .

For , we get .

Therefore, the sum of the given series is

 . **ANS.**

The sum up to infinity is,

. **ANS.**

**Problem-11:** Sum the seriesto *n* terms.

**Solution:** Here, the nth term of the given series is,







The sum of the given series is





For , we get

.



Putting the value of *c* in (1), we get



 **ANS.**

**Exercise:**

**Problem-01:** Sum the seriesto *n* terms.

**Problem-02:** Sum the seriesto *n* terms.

**Problem-03:** Sum the seriesto *n* terms.

**Problem-04:** Sum the seriesto *n* terms.

**Problem-05:** Sum the seriesto *n* terms.

**Problem-06:** Sum the seriesto *n* terms.

**Problem-07:** Sum the seriesto *n* terms.

**Problem-08:** Sum the seriesto *n* terms.